Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7305

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $\vec{\mathrm{OA}} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\vec{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. L_1 is the line that passes through A and B.

(a) Find a vector equation for L_1. [2]

(b) The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\vec{\mathrm{AB}}$. Find the value of p. [3]

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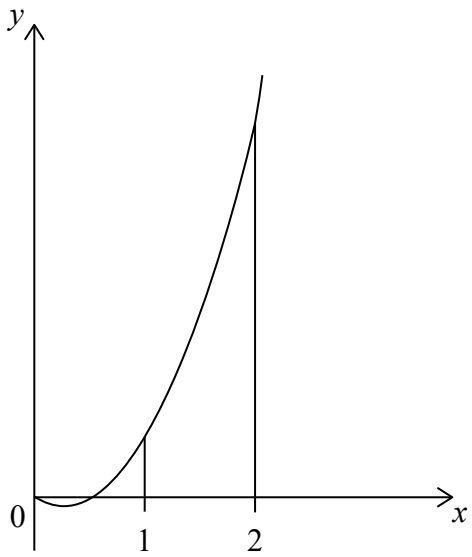
2. [Maximum mark: 6]

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



(a) Find $\int (6x^2 - 3x)\,dx$. [2]

(b) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. [4]

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3. [Maximum mark: 6]

A data set has n items. The sum of the items is 800 and the mean is 20.

(a) Find n. [2]

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance. [4]

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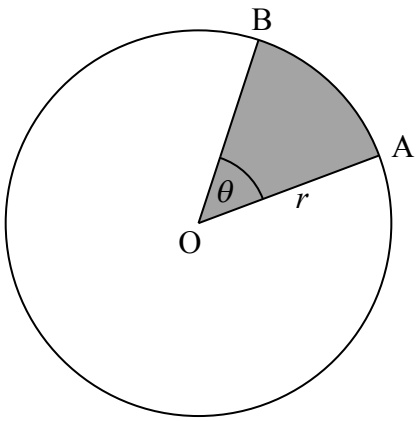
4. [Maximum mark: 7]

The following diagram shows a circle with centre O and radius r cm.

diagram not to scale



The points A and B lie on the circumference of the circle, and $\hat{AOB} = \theta$. The area of the shaded sector AOB is $12\,\text{cm}^2$ and the length of arc AB is $6\,\text{cm}$.

Find the value of r.

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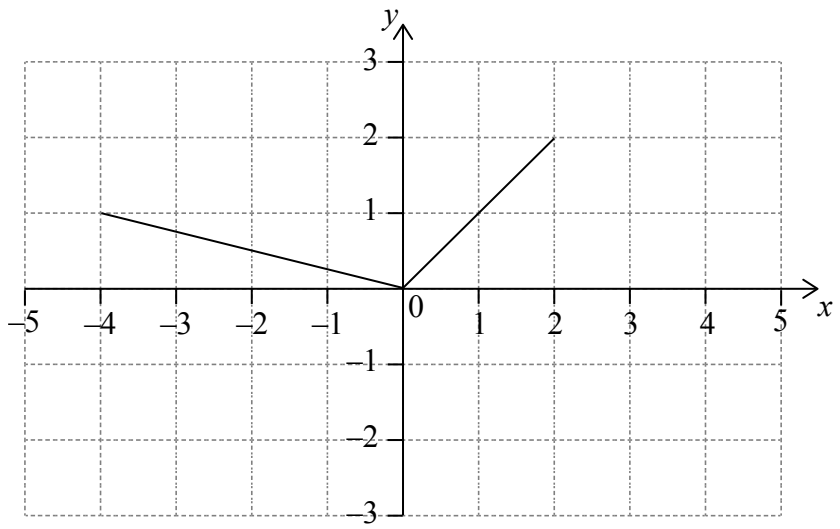
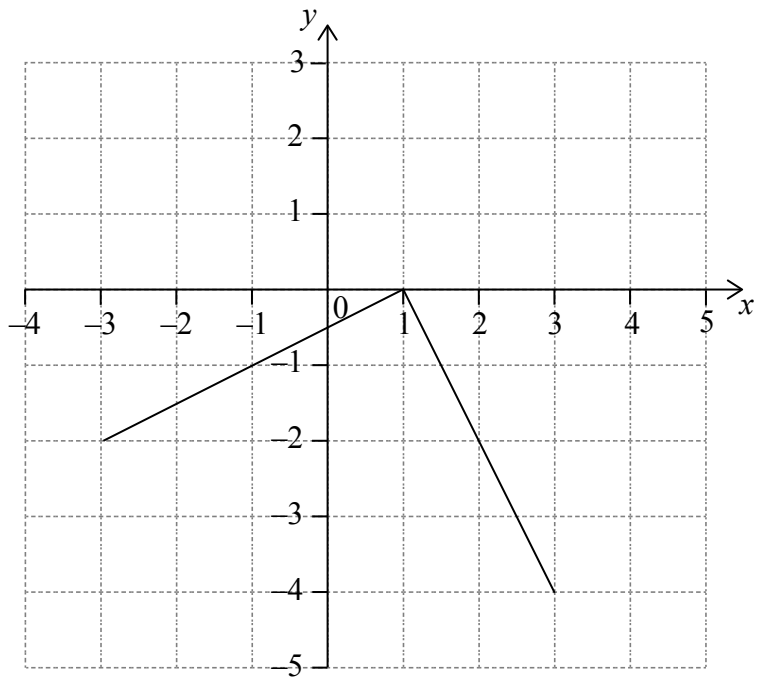
5. [Maximum mark: 6]

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, g, can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g.



Write down the value of a and of b. [4]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 7]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. Justify your answer.

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7. [Maximum mark: 8]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

(a) Show that $d = \log_c(q)$. [2]

(b) Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

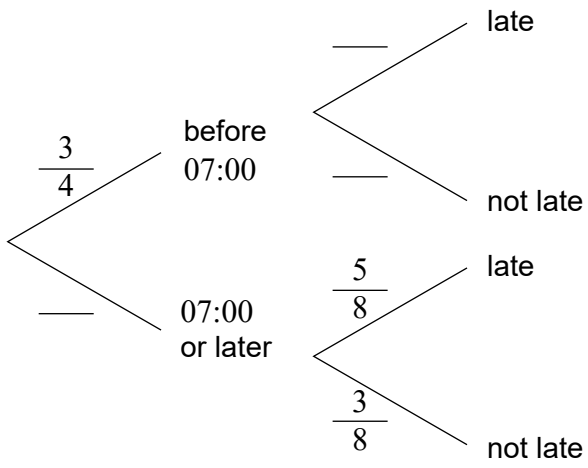
8. [Maximum mark: 14]

Pablo drives to work. The probability that he leaves home before $07{:}00$ is $\frac{3}{4}$.

If he leaves home before $07{:}00$ the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at $07{:}00$ or later the probability he will be late for work is $\frac{5}{8}$.

(a) **Copy** and complete the following tree diagram. [3]



(b) Find the probability that Pablo leaves home before $07{:}00$ and is late for work. [2]

(c) Find the probability that Pablo is late for work. [3]

(d) Given that Pablo is late for work, find the probability that he left home before $07{:}00$. [3]

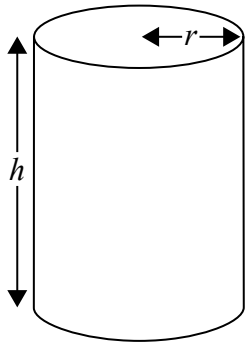
(e) Two days next week Pablo will drive to work. Find the probability that he will be late at least once. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of $20\pi \text{ cm}^3$.

diagram not to scale



(a) Express h in terms of r. [2]

The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is C.

(b) Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. [4]

(c) Given that there is a minimum value for C, find this minimum value in terms of π. [9]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$.

(a) (i) Write down $f'(2)$.

(ii) Find $f(2)$. [4]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

(b) Show that the graph of g has a gradient of 6 at P. [5]

(c) Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. Find the y-coordinate of Q. [7]